

ВИВЧЕННЯ КООРДИНАТ І ВЕКТОРІВ У 8 КЛАСІ НУШ ЗА АВТОРСЬКИМ ПІДРУЧНИКОМ ІНТЕГРОВАНОГО КУРСУ МАТЕМАТИКИ

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STUDYING COORDINATES AND VECTORS IN 8TH GRADE OF NUS ACCORDING TO THE AUTHOR'S TEXTBOOK OF THE INTEGRATED COURSE OF MATHEMATICS

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АНОТАЦІЯ

Формулювання проблеми. Реформа «Нова українська школа» (НУШ) спрямована на формування в учнів ключових компетентностей, які сприятимуть їх подальшому навчанню у вищій школі та самореалізації в майбутній професійній діяльності. Згідно з цією реформою за процес навчання в українських школах організовано наступним чином: за затвердженими Державними стандартами освіти і базовим навчальним планом, розроблено модельні навчальні програми з усіх освітніх галузей, зокрема, з математики, а вже за цими модельними програмами створюються підручники та інші засоби навчання. Метою статті є висвітлення методичних особливостей вивчення координат і векторів у 8 класі Нової української школи за підручником авторського колективу Олександр Школьний, Євген Нелін, Андрій Милиник, Юлія Простакова, створеного за модельною навчальною програмою авторського колективу на чолі з Марією Василюшин. За цією програмою і підручником вектори і координати вивчаються саме у 8 класі. Це дасть можливість вивчати тему «Геометричні перетворення площини» на початку 9 класу, а вивчення цієї теми в подальшому сприятиме більш коректному викладу матеріалу, що стосується побудови графіка квадратичної функції методом геометричних перетворень. Крім того, координати і вектори потрібні у 8 класі, зокрема, вчителям фізики для розв'язування багатьох задач цієї освітньої галузі.

Методи та матеріали. Було використано теоретичний аналіз методичної літератури, порівняльний аналіз, систематизацію та узагальнення наявних теоретичних досліджень з тематики статті, а також педагогічне спостереження та узагальнення власного педагогічного досвіду щодо навчання математики у 8 класі НУШ.

Результати. У роботі ми детально описуємо методику вивчення координат і векторів, яке передбачає вивчення наступних підрозділів: «Система координат на площині. Координати середини відрізка. Відстань між двома точками з даними координатами. Рівняння кола.», «Вектор. Модуль вектора. Колінеарні та рівні вектори. Дії над векторами.», «Координати вектора. Дії над векторами в координатній формі.», «Рівняння прямої. Взаємне розташування кола і прямої та двох кіл.», «Скалярний добуток векторів та його застосування.» Зокрема, важливим для вивчення координат вектора є введення поняття базису, який відповідає заданій системі координат на площині. Це дає можливість досить просто і традиційно для курсу математики ввести поняття координат вектора як коефіцієнтів його розкладу через вектори базису.

Висновки. Апробація підручника згаданого авторського колективу протягом 2024-2025 навчального року в навчальних закладах України показує, що учні цілком позитивно сприймають місце і спосіб вивчення координат і векторів на

ABSTRACT

Formulation of the problem. The New Ukrainian School (NUS) reform is aimed at developing key competencies in students that will facilitate their further education in higher education and self-realization in future professional activities. According to this reform, the learning process in Ukrainian schools is organized as follows: according to the approved State Education Standards and the basic educational plan, model educational programs have been developed in all educational fields, in particular, in mathematics, and textbooks and other teaching aids are created based on these model programs. The goal of the article is to highlight the methodological features of studying coordinates and vectors in the 8th grade of the New Ukrainian School according to the textbook of the author team, Oleksandr Shkolnyi, Yevhen Nelin, Andrii Mylianyk, and Yulia Prostakova, created according to the model program of the author team led by Maria Vasylushyn. According to this program and textbook, vectors and coordinates are studied in the 8th grade. This will make it possible to study the topic "Geometric transformations of the plane" at the beginning of the 9th grade, and future study of this topic will contribute to a more accurate presentation of the material on constructing a graph of a quadratic function using geometric transformations. In addition, coordinates and vectors are needed in the 8th grade, particularly for physics teachers to solve many problems in this field.

Materials and methods. Theoretical analysis of methodological literature, comparative analysis, systematization, and generalization of existing theoretical research on the topic of the article were used, as well as pedagogical observation and generalization of one's own pedagogical experience in teaching mathematics in the 8th grade of the New Ukrainian School.

Results. In the work, we describe in detail the methodology for studying coordinates and vectors, which involves studying the following subsections: "Coordinate system on the plane. Coordinates of the midpoint of a segment. Distance between two points with given coordinates. Equation of a circle", "Vector. Absolute value of a vector. Collinear and equal vectors. Operations on vectors", "Vector coordinates. Operation on vectors in coordinate form", "Equation of a line. Mutual location of a circle and a line and two circles", "Dot product of vectors and its application." In particular, it is important for studying vector coordinates to introduce the concept of a basis, which corresponds to a given coordinate system on the plane. This makes it possible to introduce the concept of vector coordinates as coefficients in an expansion using basis vectors in a fairly simple, traditional way for a mathematics course.

Conclusions. Testing of the textbook of the aforementioned author team during the 2024-2025 academic year in Ukrainian schools shows that students quite positively perceive the place and method of studying coordinates and vectors on the plane and properly use them in their educational activities. However,

площині та належним чином їх використовують у своїй навчальній діяльності. Однак, пропонувані способи опанування даного матеріалу учнями 8 класу НУШ потребує подальших досліджень. Проте ми переконані в доцільності та перспективності описаного підходу і будемо й надалі користуватися ним для формування ключових компетентностей учнів.

КЛЮЧОВІ СЛОВА: Нова українська школа; компетентнісний підхід до навчання; інтегрований курс математики; вектори і координати; методичні особливості; інноваційні методи навчання.

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the proposed method for students in 8th grade at NUS to master this material requires further research. However, we are convinced of the feasibility and prospects of the described approach and will continue to use it to develop students' key competencies.

KEYWORDS: New Ukrainian School; competency-based approach to learning; integrated course of mathematics; vectors and coordinates; methodological features; innovative teaching methods.

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INTRODUCTION

Problem statement. Analysis of current research. Currently, the education reform “New Ukrainian School” (NUS) is underway in Ukraine. It arose as a result of the discrepancy between the demands of Ukrainian society regarding the Ukrainian education system and the results of its functioning, which were finally formed by the mid-2010s. This discrepancy was manifested in the results of the state final attestation (SFA) of graduates of grades 9 and 11, the external independent assessment of the quality of knowledge (EIA) of applicants to higher education institutions (HEI), as well as in the results of international comparative studies PISA (see, for example, NUS 2024, Bakhrushyn 2019; Mazorchuk et al. 2019, Vakulenko et al. 2021, UCEQA, 2021 and others).

As we have already noted in the article (Shkolnyi 2024), the main idea of the NUS reform is the orientation of students, first of all, to successful integration into society after graduation, professional fulfillment and personal satisfaction from their own activities in the adult world. Therefore, it is natural that the theoretical basis of the NUS reform is a competency-based approach, which orients the entire process of education at school not on the formal acquisition of knowledge, skills and abilities by students in a particular subject area, but on the formation of appropriate personality characteristics in them that will allow them to ensure successful self-realization in the future. In order to implement this concept, State Education Standards were created (CMU 2018, CMU 2020, CMU 2024, MES of Ukraine 2024), as well as model educational program for each of the 9 educational branches defined in the NUS concept. For example, for the mathematical educational field, more than ten model educational programs by different authors have been approved for primary and basic secondary schools only (MES of Ukraine 2024).

Within the framework of their academic freedom, teachers, based on model programs, must develop and implement appropriate their own educational program, create calendar-thematic planning and other didactic materials for carrying out educational activities. However, it is clear that textbooks remain the main means of teaching. Currently, there are more than 10 mathematics textbooks for primary school and for basic secondary school (IMCE 2024). The team of authors led by Maria Vasylyshyn (Mariia Vasylyshyn, Andrii Mylianyk, Mykola Pratsiovytyi, Yulia Prostakova and Oleksandr Shkolnyi) proposed a model mathematics program for grades 5-6 and 7-9 (Vasylyshyn et al. 2021, Vasylyshyn et al. 2023), which is distinguished by innovative approaches to the formation of the content of educational material and methods of its study. This program is focused on European and world educational trends of recent decades. In particular, it strengthens the probabilistic-statistical line, suggests studying spatial geometric shapes in grades 7-9 in parallel with flat ones, examines the basics of financial literacy, etc. You can learn more about the features of this program by reading the article (Shkolnyi 2023).

One of the important features of the program (Vasylyshyn et al. 2023) is the transfer of the study of vectors and coordinates from grade 9 to grade 8, and the corresponding material concerning the areas of geometric figures – on the contrary, from grade 8 to grade 9. The authors saw a need for this for several reasons. First, the vector-coordinate method is significantly used not only for solving mathematical problems, but also for problems in many natural sciences, in particular, in physics. The need for appropriate educational material in grade 8 has been repeatedly expressed by physics teachers. Secondly, the study of vectors and coordinates in grade 8 makes it possible to begin studying the 9th grade mathematics course with geometric transformations of the plane, which, in turn, will make it possible to more correctly explain the construction of the graph of the quadratic function $y = ax^2 + bx + c$ by geometric transformations of the graph of the function $y = x^2$ already known to students. Indeed, for a mathematically correct explanation of constructing a graph of a quadratic function, students must already be familiar with the analytical task of parallel translation to a given vector, symmetry with respect to the abscissa axis, and deformation to the abscissa axis, which are impossible without prior experience working with vectors and coordinates on the plane. In the textbook of the integrated mathematics course for grade 8 (Shkolnyi et al. 2025) by the author team consisting of Oleksandr Shkolnyi, Yevhen Nelin, Andrii Mylianyk, and Yulia Prostakova, created according to the model program (Vasylyshyn et al. 2023), the section “Coordinates and Vectors on the Plane” is studied, which ensures the implementation of the above ideological scheme.

We should also note that there are many foreign publications on this topic (see, for example, Demetriadou & Gagatsis 2001, Guedet-Chartier 2004, Jensen et. al. 2017, Wutchana et. al. 2015 and others), but they are of little application to Ukrainian realities, since they do not take into account the peculiarities of the educational process in our country, especially during the implementation of the NUS reform. In addition, currently Ukrainian teenagers, due to martial law, are in special conditions and require a different approach to learning from the traditional ones (more details about teaching mathematics under martial law in Ukraine are described in the article Matiash & Shkolnyi 2025).

The purpose of this article is to highlight the features of studying coordinates and vectors in the 8th grade mathematics course of the New Ukrainian School. We will focus on what teachers should pay attention to when seeking to consciously develop in students of the New Ukrainian School knowledge, skills and abilities (competences) related to their mastery of the relevant educational material.

METHODS OF THE RESEARCH

To achieve the goal of the study, we use a theoretical analysis of methodological literature related to the chosen topic. Also, as an empirical method, we use surveys of teachers and students, as well as observation of the educational process in secondary schools during the testing of teaching materials in mathematics for grade 8 of the New Ukrainian School. In this article, we also operate with various methods of scientific knowledge: comparative analysis to clarify different points of view on the problem; systematization and generalization to formulate conclusions and recommendations on the formation of the competences connected to study coordinates and vectors for students of grades 7-9; we also summarize our own pedagogical experience and observations of the authors on the process of teaching mathematics in Ukrainian schools.

RESULTS OF RESEARCH

According to the educational program (Vasylyshyn et al. 2023), the study of coordinates and vectors is provided at the end of grade 8. The corresponding section of the textbook is called "Coordinates and Vectors on the Plane" and consists of the following paragraphs:

- Coordinate system on the plane. Coordinates of the midpoint of a segment. Distance between two points with given coordinates. Equation of a circle.
- Vector. Vector absolute value. Collinear and equal vectors. Operations on vectors.
- Vector coordinates. Operations on vectors in coordinate form.
- Equation of a line. Mutual position of a circle and a line and two circles.
- Scalar product of vectors and its application.

The last paragraph is studied optionally, since the relevant material is not provided for in the mentioned program, but it is traditional for other programs and textbooks for grades 7-9, both for the National Secondary School and previous years.

The paragraph "Coordinate system on the plane. Coordinates of the midpoint of a segment. Distance between two points with given coordinates. Equation of a circle." contains information about the Rectangular Cartesian Coordinate System (RCCS) on the plane and its simplest problems. In the 7th grade, students have already become familiar with the RCCS, in particular, they are already able to solve the following simplest problems:

- determining the coordinates of a given point;
- constructing a point with given coordinates.
- Further, this paragraph studies two more simplest RCCS problems:
- finding the distance between points with known coordinates (length of the segment);
- finding the coordinates of the midpoint of the segment from the known coordinates of its ends.

The motivation for studying these problems is a practical problem about the arrangement of children in a cinema hall (the dialogue between Petryk and Tetianka at the beginning of the paragraph). It is known that historically it was the idea of seating people in a theatre that led Rene Descartes to invent this coordinate system. For a convenient justification of the formula for the distance between two points in the PDSK and the formula for the coordinates of the midpoint of a segment in the RCCS, two auxiliary lemmas are proved.

Lemma 1 (on the distance between points on a coordinate line). If two different points $A(a)$ and $B(b)$ are given on some coordinate line, then $AB = |b - a|$.

Lemma 2 (on the coordinates of the midpoint of a segment on a coordinate line). If two different points $A(a)$ and $B(b)$ are given on some coordinate line and point $C(c)$ is the midpoint of the segment AB , then $c = \frac{a+b}{2}$.

Next, using these two lemmas, the following theorem is proved.

Theorem (on the simplest problems of RCCS). Let in a RCCS the points $K(x_1; y_1)$ and $M(x_2; y_2)$ are given. Let also the point $L(x_0; y_0)$ is the midpoint of the segment KM . Then 1) $KM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; 2) $x_0 = \frac{x_1+x_2}{2}$; $y_0 = \frac{y_1+y_2}{2}$.

In proving this theorem, in addition to the mentioned lemmas, the Pythagorean theorem (to prove formula 1) and the Thales theorem (to prove formula 2) are used, which were recently studied in grade 8. This additionally emphasizes the naturalness of studying this topic in grade 8, and not in grade 9. The application of the simplest RCCS problems is demonstrated immediately after proving the theorem – the educational video suggests using them to solve problems.

Another application of the simplest problem about the formula for the distance between points in the RCCS is to derive the equation of a circle. Students have already studied the equation of a line in grade 7. Therefore, after updating the concept of the graph of an equation with two variables, we can explain what it means to compose the equation of a line on a plane: composing the equation of a certain line on a plane (the GPT of the plane) means to relate the known parameters of this line to the coordinates of its arbitrary point. For instance, the equation $2x - 7y + 3 = 0$ define in a RCCS a line, where $M(x; y)$ is an arbitrary point of this line and numbers 2, -7 and 3 are their known parameters that allow to draw this line in the RCCS.

It is advisable to remind students that from the 7th grade textbook they know that to name a certain set of points of the set of the GMT plane, which is given by some characteristic property, it is necessary to prove the following two statements:

- each point of the given set has this characteristic property;
- all points that have this characteristic property belong to the given set.

Therefore, to assert that an equation with two variables x and y is an equation of a certain line on the coordinate plane xOy (GPT of plane), it is necessary to make sure:

- if the point belongs to the given line, then its coordinates are a solution of the considered equation;
- any solution $(x; y)$ of the considered equation are the coordinates of a point that belongs to the given line.

Next, the canonical equation of a circle centered at the point $M_0(x_0; y_0)$ and radius R is derived. It has the following form $(x - x_0)^2 + (y - y_0)^2 = R^2$. When considering the relevant justification, students should be aware that the textbook shows: if the point $M(x; y)$ is an arbitrary point on a given circle, then $\sqrt{(x - x_0)^2 + (y - y_0)^2} = R$ (*). But if the coordinates of the point $M(x; y)$ satisfy this equation, then we can conclude that $M_0M = R$, and this means that $(x; y)$ are the coordinates of the point M , which belongs to the circle. So, equation (*) is indeed the equation of a circle, which is further transformed (by equivalent transformations) into the canonical equation $(x - x_0)^2 + (y - y_0)^2 = R^2$. After that, the textbook examines examples of constructing the canonical equation of a circle and determining the coordinates of the center and radius of a circle given its canonical equation.

A separate interesting problem is to determine the coordinates of the center and radius of a circle given by an equation of the form $x^2 + y^2 + ax + by + c = 0$. It is worth emphasizing that such an equation can either define a circle or not, giving appropriate examples. We suggest considering the analytical conditions under which an equation of this type will define a circle in the classes of the mathematics club for inquisitive students. For such students, the material from section "Grandpa Taras's curiosities" for this paragraph, which concerns polar coordinate systems on the plane and in space and their use in geography and astronomy, will also be useful.

In the paragraph "Vector. Vector modulus. Collinear and equal vectors. Actions on vectors." the concept of a vector, vector modulus, and also actions on vectors (addition, subtraction, and multiplication by a number) are considered. After the motivational block, where Petryk and Tetianka discuss the use of directed segments (arrows) to indicate movement, the traditional definition of a vector is given: "A segment, which indicates which of the points is the initial and which is the final, is called a directed segment or vector." Next, the definitions of the vector modulus, collinear vectors (in particular, co-directed and multi-directed) and equal vectors are introduced. After this, the property of vector offset is formulated and proved: "Whatever the vector \vec{a} is, it can always be offset in a unique way from any point in the plane."

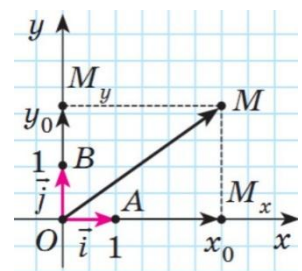
Further in the paragraph the definitions of the sum of vectors according to the triangle rule and the parallelogram rule are introduced. Both of these rules are equal, the justification for this is also given in the textbook. The rule for adding two vectors is further generalized to the case of adding several vectors, in particular, to the case when vectors, postponed "one by one" form a closed polyline. In the latter case, it is natural to consider the sum of these vectors as a vector whose beginning coincides with its end, that is, the zero vector. The definition of the difference of vectors is given by analogy with the definition of the difference of numbers. After this, the rule for subtracting two vectors is given and justified: to subtract two vectors, you need to postpone them from one point, then connect the ends of the postponed vectors and point the arrow to the vector from which you are subtracting. For a better understanding of the subtleties of performing addition and subtraction of vectors, an educational video is provided, which considers specific examples of performing these two operations on vectors.

At the end of the paragraph, the definition of the product of a vector by a number is introduced and the execution of this action is illustrated in combination with the actions of adding and subtracting vectors in another educational video. It is important for further applications to substantiate the fact that of two collinear vectors, one can always be expressed in terms of the other. The textbook provides an algorithm for determining the coefficient λ , for which $\vec{b} = \lambda \cdot \vec{a}$, if it's known that $\vec{a} \parallel \vec{b}$. Next, an example is considered of how, under certain conditions, one given vector can be expressed in terms of two other given vectors, where the algorithm just mentioned is applied. The paragraph concludes with a theorem on the properties of vectors, the proof of which is given in a separate video. We recommend requiring the assimilation of the material of the last video with the proof of the properties of vectors only for students who are studying at a sufficient and high level of educational achievements; for the rest of the students, it is enough to simply know the properties of actions on vectors and be able to use them.

Inquisitive students are advised to pay attention to the interesting facts from the section "Grandpa Taras's curiosities" for this paragraph, which emphasizes that vectors in physics and mathematics are, generally speaking, different objects: mathematical vectors are "free" due to the property of postponement, while for vectors in physics, not only the magnitude of the vector and its direction are important, but also the point of postponement, that is, they are "tied".

The paragraph "Vector coordinates. Actions on vectors in coordinate form." is devoted to the study of vector coordinates and actions on vectors in coordinate form. Here we introduce the concept of a basis associated with a given RCCS, the radius vector of a given point and relate the coordinates of a point to the expansion of the radius vector of this point in terms of the basis vectors. We present the corresponding fragment of the theoretical material of the textbook (Shkolnyi et al. 2025), because it is not traditional and is not found in other Ukrainian textbooks.

«Let's consider in a RCCS the points $O(0; 0)$, $A(1; 0)$ and $B(0; 1)$. Let's denote $\vec{OA} = \vec{i}$, $\vec{OB} = \vec{j}$ (see the picture). Ordered set of vectors $(\vec{i}; \vec{j})$ is called the basis that correspond given RCCS. Let's consider also in the same RCCS an arbitrary point $M(x_0; y_0)$, which does not belong to any of the coordinate axes, and the points $M_x(x_0; 0)$, $M_y(0; y_0)$. The vector \vec{OM} is called radius-vector of the point M . According to the parallelogram rule $\vec{OM} = \vec{OM}_x + \vec{OM}_y$. Because $\vec{OM}_x \parallel \vec{i}$ and $\vec{OM}_y \parallel \vec{j}$, then $\vec{OM}_x = x_0 \cdot \vec{i}$ and $\vec{OM}_y = y_0 \cdot \vec{j}$. Therefore, $\vec{OM} = x_0 \cdot \vec{i} + y_0 \cdot \vec{j}$. If the point $M(x_0; y_0)$ belongs to one of coordinate axes, then $x_0 = 0$ or $y_0 = 0$. If $x_0 = 0$, then $\vec{OM} = 0 \cdot \vec{i} + y_0 \cdot \vec{j}$. If $y_0 = 0$, then $\vec{OM} = x_0 \cdot \vec{i} + 0 \cdot \vec{j}$. So, anyway the radius-vector OM of a point $M(x_0; y_0)$ can be expand through the vectors of the basis $(\vec{i}; \vec{j})$, where coordinates of this point are the coefficient of this expansion.»



This interpretation of the coordinates of a point makes it possible to give a completely correct definition of the coordinates of a vector (the coordinates of a vector in a certain RCCS on a plane are the coefficients of the expansion of this vector through the basis vectors $(\vec{i}; \vec{j})$, which corresponds to this RCCS), and also to prove the rule for finding the coordinates of

a vector from the known coordinates of its beginning and end: to find the coordinates of a vector, it is necessary to subtract the corresponding coordinates of its beginning from the coordinates of its end. It is also advisable to draw the attention of students to the fact that the definition of the coordinates of a vector implies: if the beginning of a vector is the origin of coordinates, then the coordinates of the vector coincide with the coordinates of its end.

An example of the use of coordinates in everyday life problems is the use of these coordinates to determine the location of children in the cinema hall during a movie screening (see the dialogue between Petryk and Tetianka at the beginning of the paragraph). To solve this problem, it is fundamental that equal vectors have the same coordinates and vice versa - if vectors have equal coordinates, then they are equal. This fact is also substantiated in the textbook. It is also convenient to use vector coordinates in mathematical problems, in particular, to determine the coordinates of vectors that are formed as a result of actions on given vectors. For this purpose, the educational video, which is mandatory for all students to study, proves the rules for actions on vectors in coordinate form. The use of these rules is illustrated by a corresponding example at the end of the paragraph.

It is also important to emphasize that if two vectors are collinear, then their coordinates are proportional. This rule will be often used in the future when solving many problems. The textbook also suggests that students independently verify that the correct and converse statement is: if the coordinates of two vectors are proportional, then the vectors are collinear. This is really easy to do independently, using the formula for calculating the product of a vector by a number in coordinate form and the fact that the result of multiplying any vector by a number is a vector collinear to it. The paragraph concludes with a proof of the formula for the length of a vector by its known coordinates. In this case, the formula for the distance between two points and the formula for finding the coordinates of a vector by the known coordinates of its beginning and end are used. Curious students can also learn more from the section "Grandpa Taras's curiosities" about the affine coordinate system and its main problems.

In the paragraph "Equation of a straight line. Mutual location of a circle and a straight line and two circles" the equation of a straight line, as well as the mutual location of a straight line and two circles, is studied. In the motivational block to the paragraph, a specific example shows that a straight line (the trajectory of an airplane) can be defined by a point and a vector that is collinear to this straight line (i.e., is parallel to it or belongs to it). In general, there are three main ways to specify a straight line on a plane:

- through a given point on a line and a vector parallel to this line;
- through two given points on this line;
- through a given point on a line and a vector perpendicular to this line.

In the paragraph, the corresponding equation of the line is compiled for the first two methods:

- $\frac{x-x_0}{m} = \frac{y-y_0}{p}$ - equation of a line passing through a point $M_0(x_0; y_0)$ and has a direction vector $\vec{s}(m; p)$;
- $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$ - equation of a line passing through two points $M_1(x_1; y_1)$ i $M_2(x_2; y_2)$ of this line.

The equation of the line for the last method of assignment is given in the last paragraph of the section, which is studied optionally, that is, if there is a reserve of study time (there is no corresponding material in the model program Vasylyshyn et al. 2023).

The following example shows how to reduce each of the given equations to the form $ax + by + c = 0$ and states without proof that any equation of a line can be reduced to this form. An equation of this form is called a general equation of a line. Curious students can get acquainted with the proof of this fact in the lessons of the mathematics club.

The second part of the paragraph is devoted to methods for establishing the relative position of a circle and a line and two circles on a plane using the equations of a circle and a line. Students already know from the 7th grade mathematics course that there are 3 types of relative positions of a circle and a line on a plane:

- a line is a secant of a circle;
- a line is tangent to a circle;
- a line has no points in common with a circle.

Since both a circle and a line can be given by equations, it is natural to determine their mutual location analytically, using these equations. The mutual location of a circle and a line can be determined by the number of solutions of the system consisting of the equation of a circle and the equation of a line. This system is reduced by substitution to a quadratic equation, which students already know how to solve. Note that to establish the mutual location of a circle and a line, it is not necessary to find the solutions themselves; it is enough to find the discriminant of the quadratic equation obtained after substitution and compare it with zero. However, for curious students, you can also find the points of intersection of the circle and the line (if they exist).

Note that the method just described is somewhat cumbersome, so Grandpa Taras below also offers an alternative method of determining the relative position of a circle and a line. He says that in high school they prove the formula for the distance from a point $M_0(x_0; y_0)$ to the line $ax + by + c = 0$: $h = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$. Using this formula, we can find the corresponding distance, then compare it to the radius of the circle and draw a conclusion. Both methods of establishing the relative position of a circle and a line are illustrated with examples.

Also from the 7th grade mathematics course, students know that there are 5 types of mutual arrangement of two circles:

- circles have external contact;
- circles have internal contact;
- circles have two common points;
- circles have no common points and one of them is inside a flat circle bounded by another;
- circles have no common points and none of them is inside a flat bounded by another.

The analytical method of establishing their mutual location requires determining the coordinates of the centers and radii of both circles according to their equations, as well as calculating the distance between their centers according to the formula for the length of the segment in the PDSK. After that, this distance should be compared with the sum of the radii and the modulus of their difference and draw the appropriate conclusion. An example of establishing the mutual location of two circles

using an analytical method is also given in the paragraph. The material of the paragraph is completed by an educational video on using a graphical calculator to establish the mutual location of a circle and a line and two circles. This video is mandatory for all students to study. Also, from the section "Grandpa Taras's curiosities" of the paragraph, interested students can learn about two more equations of a line - in segments on the axes and parametric.

The paragraph "Scalar product of vectors and its application" is studied optionally. The motivation for studying the scalar product is the determination of the work of a constant force (applied by Tetianka while transporting vegetables in the country) on moving a cart at an angle to the direction of movement of this cart. According to the formula from the physics course, the work A performed by a constant force \vec{F} when moving a body from point A to point B , that is, by the vector \vec{AB} , calculate by the formula: $A = |\vec{F}| \cdot |\vec{AB}| \cdot \cos\varphi$, where φ is the angle between the direction of the force and the direction of movement. Thus, the result of the action on two vectors \vec{F} i \vec{AB} is a number A . Such an action on vectors is atypical and requires separate study.

To correctly define the scalar product of vectors in paragraph 28, first introduce the definition of trigonometric functions of an angle whose degree measure is in the range from 0 to 180 degrees. The method of this introduction is traditional, using a single semicircle. After that, introduce the definition of the scalar product of vectors given by their coordinates in the RCCS (Shkolnyi et al. 2025): «The scalar product of the non-zero vectors $\vec{a}(a_1; a_2)$ and $\vec{b}(b_1; b_2)$ is called the number $a_1b_1 + a_2b_2$. The dot product of vectors is denoted by $\vec{a} \cdot \vec{b}$, that is $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2$.» Further, based on this definition, the properties of the scalar product of vectors are formulated and proven. It is also advisable to draw students' attention to the fact that the scalar product of a vector \vec{a} on the vector \vec{a} (that is, the product $\vec{a} \cdot \vec{a}$) is denoted by \vec{a}^2 and is called the scalar square of the vector \vec{a} . Then, by the definition of the dot product, it is easy to see that the scalar square of a vector is equal to the square of its length.

Finally, after all the preparatory work, we can proceed to the answer to the question of the motivational block regarding the application of the scalar product of vectors in practice. To do this, first the definition of the angle between two vectors is introduced, and then the following theorem is formulated and proved in the educational video: "The scalar product of non-zero vectors is equal to the product of their lengths by the cosine of the angle between them." From this it follows that the work A performed by a constant force \vec{F} when moving a body from point A to point B , can be written as the scalar product of these vectors: $A = \vec{F} \cdot \vec{AB}$.

At the end of the section, mathematical applications of the scalar product of vectors are considered for finding the angle between given vectors and deriving the equation of a line passing through a point $A(x_0; y_0)$ of the line perpendicular to a given vector (normal vector) $\vec{n}(a; b)$. This equation has the form $a(x - x_0) + b(y - y_0) = 0$ and it can also be reduced by simple transformations to the general equation of a straight line $ax + by + c = 0$. It is also worth noting to curious students that the coefficients of the general equation of any line have a geometric meaning - they are the coordinates of the normal vector of this line.

CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

In our opinion, the above-described study of coordinates and vectors on the plane will contribute to the achievement of both goals set by the authors of the program (Vasylyshyn et al. 2023) and the textbook (Shkolnyi et al. 2025):

- 1) teachers of physics and other natural sciences will have a tool for solving problems in their educational field using the vector-coordinate method;
- 2) mathematics teachers will have the opportunity in the 9th grade to mathematically correctly introduce the concept of geometric transformation of the plane and its analytical task and, based on the method of geometric transformations, also quite correctly give an algorithm for constructing a graph of a general quadratic function.

Piloting and testing of the model program (Vasylyshyn et al. 2023) and the textbook (Shkolnyi et al. 2025) during the 2024-2025 academic year in educational institutions of Ukraine shows that students quite positively perceive such a place and such a method of studying coordinates and vectors on the plane and use them appropriately both in mathematics and in other educational fields. However, we note that a thorough study of the effectiveness of such a method of mastering this material by students of the 8th grade of the National School is still ahead, since the aforementioned program and textbook are used in the educational process, in fact, for only one academic year. However, we are convinced of the feasibility and prospects of the described approach and will continue to use it to form key competencies of students in the context of the "New Ukrainian School" reform.

CONFLICT OF INTEREST

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DATA AVAILABILITY

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