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ТЕОРЕМИ ПРО КІЛЬКІСТЬ КОРЕНІВ КУБІЧНОГО РІВНЯННЯ ТА ЇХ РОЗТАШУВАННЯ ЯК ЗАСІБ РОЗВИТКУ НАОЧНОГО МИСЛЕННЯ УЧНІВ

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АНОТАЦІЯ

Постановка проблеми. *Основою навчання математики є логічне мислення, засноване на роботі лівої півкулі. У науково-методичних дослідженнях збільшується обсяг робіт, пов'язаних з питанням організації навчання шляхом координації роботи як лівої, так і правої півкуль, тобто розвитку поряд з логічним і інших видів мислення, особливо наочно-образного. Розроблено рекомендації щодо методики розвитку наочно-образного мислення учнів на уроках математики. Проте при вивченні основ алгебри та аналізу актуальними є також удосконалення методики розвитку наочно-образного мислення та розробка методичних матеріалів для позакласної роботи.*

Матеріали та методи. *Матеріалом дослідження є педагогічна, методична література, досвід зарубіжних та вітчизняних учених. У процесі дослідження використовувалися емпіричні методи (спостереження, перевірка, експеримент), загальнонаукові методи (аналіз, синтез, конкретизація, систематизація, узагальнення). Для доведення теорем використано метод зворотного доведення.*

Результати. *Вивчення графіків кубічної функції допомагає будувати і доводити гіпотези про кількість дійсних коренів кубічного рівняння та їх розташування, дає змогу наочно продемонструвати використання наочно-образного мислення.*

Висновки. *Навчальний матеріал про розташування коренів кубічного рівняння допомагає розвивати наочно-образне мислення учнів, формулювати наочні завдання для учнів. Ці наочні завдання слугують засобом організації математичної діяльності учнів. Це допомагає читачам зрозуміти, як створюються теореми та як проводити доведення. Він також показує залежність між дискримінантом кубічного рівняння та добутком екстремальних значень відповідної кубічної функції. Вивчати розташування коренів кубічного рівняння рекомендуємо старшокласникам на заняттях гуртка з математики.*

КЛЮЧОВІ СЛОВА: візуальне мислення; візуальна задача; візуальний пошук; кубічне рівняння; кубічна функція; похідна функції.

THEOREMS ON THE NUMBER OF ROOTS OF A CUBIC EQUATION AND THEIR LOCATION AS A MEANS OF DEVELOPING STUDENTS' VISUAL THINKING

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ABSTRACT

Formulation of the problem. *The basis of teaching mathematics is logical thinking, (which is associated with) based on the work of the left hemisphere. In scientific and methodological research, the volume of work related to the issue of organizing learning by coordinating the work of both the left and right hemispheres is increasing, that is, the development of other types of thinking, especially visual, along with logical thinking. Proposals have been developed on the methodology for the development of visual thinking of students in mathematics lessons. However, when studying the basics of algebra and analysis, improving the methodology for developing visual thinking and developing teaching materials for extracurricular activities are also urgent tasks.*

Materials and methods. *The research materials are pedagogical, methodical literature, experience of foreign and domestic scientists. In the process of research, empirical methods (observation, verification, experiment), general scientific methods (analysis, synthesis, concretization, systematization, generalization) were used. The method of reverse proof was used to prove the theorems.*

Results. *The study of graphs of a cubic function helps to build and prove hypotheses about the number of real roots of a cubic equation and their location, and makes it possible to clearly demonstrate the use of visual thinking.*

Conclusions. *Educational material on the location of the roots of a cubic equation helps to develop the visual thinking of students, to formulate visual tasks for students. These visual tasks serve as a means of organizing the mathematical activity of students. It helps readers understand how theorems are created and how to look for proofs. It also shows the relationship between the discriminant of a cubic equation and the product of the extreme values of the corresponding cubic function. We recommend studying the location of the roots of the cubic equation for high school students in maths club training.*

KEYWORDS: visual thinking; visual problem; visual search; cubic equation; cubic function; function derivative.

INTRODUCTION.

One of the main goals of teaching mathematics, in particular teaching the algebra and fundamentals of analysis, is the formation and development of students' thinking. In order to successfully live in the information age, a person needs a well-developed mindset. Until now, much attention has been paid to the development of logical thinking in schoolchildren. However, in recent years, in pedagogical and methodological studies, it has been noted that the issue of developing other types of thinking in students and ensuring the relationship of these types of thinking is relevant. One type of thinking is visual thinking.

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Abdiyeva, Sh., & Turgunbaev, R. (2023). Theorems on the number of roots of a cubic equation and their location as a means of developing students' visual thinking. *Fizyko-matematychna osvita – Physical and Mathematical Education*, 38(4), 7-13. https://doi.org/10.31110/2413-1571-2023-038-4-001

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This type of thinking is widely used in solving mathematical problems, especially problems related to the basics of analysis. However, the issue of the formation and development of visual thinking of students with the help of such questions has been little studied.

The practice of teaching mathematics to students shows that the main support of a student is logical thinking, it means, he relies on the work of the left hemisphere of the brain (Bragina & Dobrokhotova, 1988).

In recent years, a number of studies have been carried out on the effective use of visual thinking in the educational process. For example, D.Bezugly (2014) found that the methods of visual presentation of educational information in mathematics lessons provide a solution to a number of pedagogical problems, such as ensuring the intensity of learning, enhancing educational and cognitive activity, the formation and development of critical and visual thinking, visual perception (Bezugly, 2014).

In the studies of A.V. Firer (2018), the issues of developing the universal educational activity of primary school students using visualization tools are considered. Visualized tasks and visualized models have been developed as a means of studying the functional lines of algebra, their types have been identified (Firer, 2018)

However, the issues of teaching students to use visual tools when formulating mathematical concepts and theorems, especially when putting forward and proving these hypotheses, and developing visual thinking on this basis, were not under consideration.

There is also the problem of demonstrating to students the idea of the great K. Gauss that “Mathematics is a science that is studied not only by ear, but also by eye” (Dalinger, 2006).

THEORETICAL BASES OF THE RESEARCH

Currently, the term visual thinking is widely used, according to R.Arnheim (Arnheim, 1981), term means “thinking under the influence of visual processes”.

Visual thinking is an activity that ensures the creation of images, the impact on them, the reconstruction of images in an arbitrary or given direction, the use of various calculus systems when building an image, the selection of various signs and properties that are important for a person in an image.

V. P. Zinchenko and N. Yu. Vergilis (1969) define the concept of visual thinking as human activity, the product of which is the birth of new images, the creation of new visual forms that convey a certain meaning and make knowledge visible.

According to N.A. Reznik (2000), relying only on the work of the left hemisphere of the brain when teaching mathematics strains it and causes exhaustion. To prevent this, it is good to rely more on the work of the right hemisphere in mathematical education, that is, to pay more attention to visualization. To do this, he emphasizes the need to create a visual learning environment - new ways of communicating information and creating a new type of learning environment.

V.A. Dallinger proposes to build the process of teaching mathematics on the basis of a cognitive-visual (visual-cognitive) approach to the formation of knowledge, skills and abilities, which allows you to make the most of the potential of visual thinking. One of the main provisions of this approach is the wide and purposeful use of the cognitive function of visibility. The implementation of the cognitive-visual approach in the process of teaching students mathematics allows you to design a visual learning environment - a set of learning conditions in which the emphasis is encourages the use of the student's visual thinking reserves. These conditions suggest the presence of both traditional visual aids and special tools and techniques that allow you to activate the work of visual thinking.

In the implementation of the cognitive-visual approach to teaching mathematics, visualized tasks play an important role.

A visualized task is understood as a task in which the image is explicitly or implicitly involved in the condition, response, sets the method for solving the problem, creates support for each stage of the solution for giving either explicitly or implicitly accompanies at certain stages of its solution. The purpose of visualized tasks is the formation of a visual image that helps to solve emerging problems.

The use of visualized tasks helps to organize search, in particular, visual search learning activities of students.

Visual search is the process of generating new images, new visual forms that carry a specific visual-logical load and make visible the value of the desired object or its light. The starting position of such a process is the stock of ready-made visual images known to the student, the structure and elements of information, visually observable connections between them. Visualized tasks serve as a means of forming visual search skills (Dallinger, 2006).

PURPOSE OF THE STUDY

To substantiate the manifestation of visual thinking in the process of solving the problem of the number and location of the roots of a cubic equation and the possibility of using this problem in organizing the visual-search activity of students.

METHODS OF THE RESEARCH

Educational materials on polynomials, functions and their properties, pedagogical and methodical literature, experience of foreign and domestic scientists. In the process of research, empirical methods (observation, interviews), general scientific methods (analysis, synthesis, concretization, systematization, generalization), methods of mathematical analysis were used.

RESULTS OF RESEARCH AND DISCUSSION

It is known that the problem of the location of the roots of a quadratic equation is solved by its coefficients (Turgunbayev & Yoldoshev, 2004). A similar problem can be formulated for cubic equations:

How are the number of roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$ and their location related to the coefficients of this equation?

We know that a cubic equation cannot always be solved in radicals. High school students know some rational equations, in particular, how to find the roots of a cubic equation using the divisors of the free term (Zaitov et al., 2022), write equations based on their roots, the concepts of monotonicity of a function, extremum points, extremum, and how to solve the equation graphically. Students learn to check a function using a derivative in the 11th grade algebra course (Mirzaakhmedov et al., 2018). Based on this knowledge of students, the process of learning to solve the above problem can be organized as follows.

At step 1, the teacher must provide information about the number of real roots of cubic equations, because this information is not provided at school. The result of this step should be that the cubic equation has only one real root, or three real roots (at least one of which is different from the others), or two repeted roots.

In step 2, the teacher asks students to classify cubic equations by the number and location of their roots (relative to zero). It is desirable to express the classification result in the form of a table.

In step 3, the teacher asks students to write cubic equations according to the above classification.

In step 4, the teacher asks students to draw a graph of the functions corresponding to the equations obtained in the previous step using the GeoGebra or Desmos graphing calculator programs.

At step 5, the corresponding graphs are analyzed for the case when the main coefficient of the cubic equation is positive.

For clarity, let's assume that the equation $ax^3 + bx^2 + cx + d = 0 \ \ (a > 0)$ has a single positive root (**case 1**). The graph of the functions corresponding to this equation will be similar to the graphs in Fig. 1, 2, 3.

Graphs are analyzed together with students, and the following hypothesis is put forward:

For the equation $ax^3 + bx^2 + cx + d = 0 \ \ (a > 0)$ to have a single positive root, it is sufficient that the corresponding function $f(x) = ax^3 + bx^2 + cx + d$ is strictly increasing and d <0; or the product of the extreme values of the function $f(x) = ax^3 + bx^2 + cx + d$ was positive.

Step 6 explores how this assumption can be related to the coefficients of the equation. To do this, it is proposed to use the relationship between monotonicity, extrema and derivatives of the function.

Equating the derivative of the cubic function $f(x) = ax^3 + bx^2 + cx + d$ to 0, we get the following results:

$f'(x) = (ax^3 + bx^2 + cx + d)' = 3ax^2 + 2bx + c.$

$3ax^2 + 2bx + c = 0.$

Then the discriminant of the resulting quadratic trinomial is: $\Delta = b^2 - 3ac$.

If $b^2 - 3ac \le 0$, as in Figure 1, the function does not have extremum values, that is, the function is strictly increasing in $(-\infty; +\infty)$.

If $b^2 - 3ac > 0$, as shown in Figure 2, the product of the extremum values of the function is positive.

It is also clear that $f(0) = a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d = d < 0$. The above conditions are not only sufficient, but also a necessary condition for the considered equation to have one positive real root.

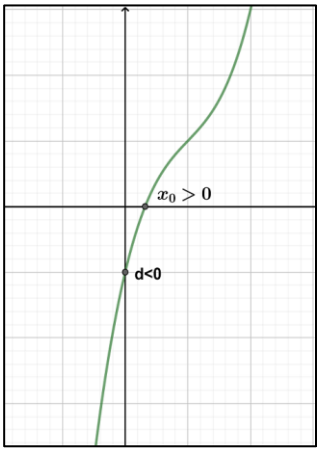

Figure 1

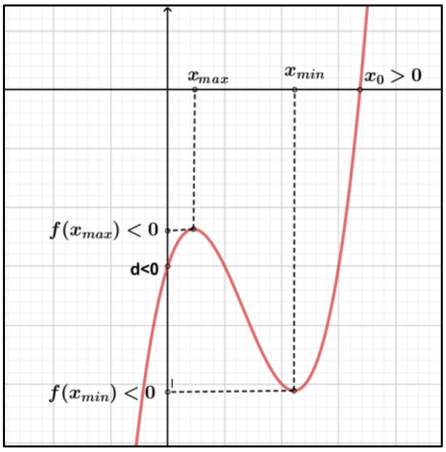

Figure 2

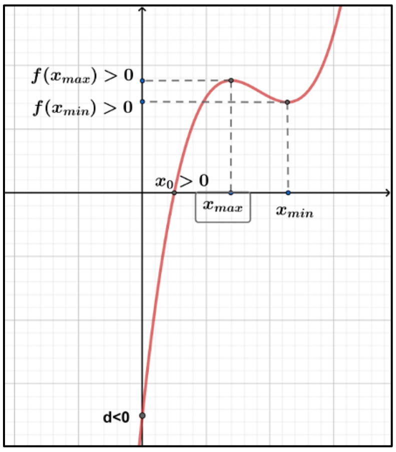

Figure 3

In step 7, the following theorem is formulated and proved.

Theorem 1. For the cubic equation $ax^3 + bx^2 + cx + d = 0 \ (a > 0)$ to have one positive real root it is necessary and sufficient that $b^2 - 3ac \le 0, d < 0$ or $b^2 - 3ac > 0, d < 0$ and the product of extremum values be positive.

We use the following lemmas to prove this theorem:

Lemma 1. If the function $f(x)$:

1) continuous in $[a; +\infty)$;

2) $\lim\limits_{x \to +\infty} f(x) = +\infty$;

3) If $(a) < 0$, then there exists a point c in the interval $[a; +\infty)$ such that $f(c) = 0$.

Proof: The function f(x) is continuous in $[a; +\infty)$ and from condition 2 it is not bounded from above. For any arbitrary positive number E, there exists a point x' in the interval $[a; +\infty)$ such that the inequality $f(x') > E$ holds.

Therefore, $f(a) < 0, \ f(x') > 0$ in the section $[a; x']$, and according to the 1st theorem of Bolzano-Cauchy (Fikhtengolts, 1955), for the given function $\exists c \in [a; x']$ point is found and $f(c) = 0$. The lemma is proved.

The following lemmas can be proved in the same way as above.

Lemma 2. If the function $f(x)$:

1) continuous in ;

2) $f(a) > 0$;

3) If $\lim_{x\to-\infty} f(x) = -\infty$ then there exists a point *c* in the interval such that $f(c) = 0$.

Lemma 3. If the function $f(x)$:

1) continuous in $(-\infty; +\infty)$;

2) $\lim_{x\to-\infty} f(x) = -\infty$ ($\lim_{x\to-\infty} f(x) = +\infty$);

If $\lim_{x\to+\infty} f(x) = +\infty$) (in line $\lim_{x\to+\infty} f(x) = -\infty$) then there exists a point *c* in the interval $(-\infty; +\infty)$ such that $f(c) = 0$.

Clearly, if the function $f(x)$ in the above theorems is strictly monotonic, then the point *c* is unique.

Proof of the theorem: First we prove the necessity of the given conditions, i.e. $ax^3 + bx^2 + cx + d = 0$ $(a > 0)$ from the existence of a single real root $x_0 > 0$ of the cubic equation $b^2 - 3ac \leq 0$, $d < 0$ or $b^2 - 3ac > 0$, $d < 0$, we show that the product of the extremum values of the function is positive.

The derivative of the function $f(x) = ax^3 + bx^2 + cx + d$ $(a > 0)$ is $f'(x) = 3ax^2 + 2bx + c$ and the discriminant of the quadratic trinomial is equal to $\Delta = b^2 - 3ac$. It can be $\Delta \leq 0$ or $\Delta > 0$.

If $\Delta \leq 0$, then the function $f(x)$ has no extrema. Since $a > 0$, the function $f(x)$ is monotonically increasing in $(-\infty; +\infty)$. From the fact that $x_0 > 0$, it follows that $0 = f(x_0) > f(0) = d$, that is, $d < 0$ (Figure 1).

Now let $\Delta > 0$. Then the square trinomial $3ax^2 + 2bx + c$ has roots $x_{max,min} = \frac{-b\pm\sqrt{b^2-4ac}}{3a}$. Let $x_{max} < x_{min}$ for precision. Clearly, the function $f(x)$ is increasing in the interval $(-\infty; x_{max})$, $(x_{min}; +\infty)$ and decreasing in the interval $(x_{max}; x_{min})$. It follows that the relationship $f(x_{max}) > f(x_{min})$ is appropriate for the extrema values of the function at the points x_1 and x_2. To prove the above assertion, we assume the converse: let $f(x_{max}) > 0$, $f(x_{min}) \leq 0$. Then the interval has one zero according to Lemma 2. Since the function is increasing in this interval, the zero of the function is unique. According to the Bolzano-Cauchy theorem, the intersection $[x_{max}; x_{min}]$ has a zero of the function. This contradicts the condition of the theorem (unique contrary to the existence of a positive root). $x_0 > 0$ means that $0 = f(x_0) > f(0) = d$, i.e., $d < 0$.

Now we prove the sufficiency, since $b^2 - 3ac \leq 0$, $d < 0$ or $b^2 - 3ac > 0$, $d < 0$ and product of function $f(x) = ax^3 + bx^2 + cx + d$ extrema values are positive we show that the given equation has one positive real root.

If $b^2 - 3ac \leq 0$, then the function $f(x)$ does not have an extremum, this function satisfies the conditions of lemma 3, so x_0 is found in the interval $(-\infty; +\infty)$ and $f(x_0) = 0$. The function $f(x)$ is increasing in the interval $(-\infty; +\infty)$, so the root is unique. It is positive because the function is increasing and $f(0) = d < 0$.

If $b^2 - 3ac > 0$, then the square trinomial $3ax^2 + 2bx + c$ has roots $x_{max,min} = \frac{-b\pm\sqrt{b^2-4ac}}{3a}$. Let $x_{max} < x_{min}$ for precision. Clearly, the function $f(x)$ is increasing in the intervals $(-\infty; x_{max})$, $(x_{min}; +\infty)$ and decreasing in the interval $(x_{max}; x_{min})$. It follows that the relation $f(x_{max}) > f(x_{min})$ is valid for the extrema values of the function at the points x_{max} and x_{min}. Let $f(x_{max}) \cdot f(x_{min}) >$. Then $f(x_{max}) > f(x_{min}) > 0$ or $0 > f(x_{max}) > f(x_{min})$. Suppose $f(x_{max}) > f(x_{min}) > 0$. Since the function $f(x)$ is monotonically increasing in the interval , $x_{max} > 0$ follows from the relation $d = f(0) < 0$. Then the conditions of the Bolsano-Cauchy theorem are fulfilled in the section $[0; x_{max}]$. The equation has one positive real root (Figure 2).

If $0 > f(x_{max}) > f(x_{min})$, the function is increasing in and from the fulfillment of conditions of lemma 1, it follows that the equation $ax^3 + bx^2 + cx + d = 0$ has one positive real root originates (Figure 3). **The theorem was proved.**

In the next steps, other theorems corresponding to the number and location of the roots of the cubic equation are formulated. We describe the formulation of these theorems using visual thinking below.

Case 2. Now let's look at the graphs of the case where the cubic equation $ax^3 + bx^2 + cx + d = 0$) has the only negative real root (Figures 4-5-6).

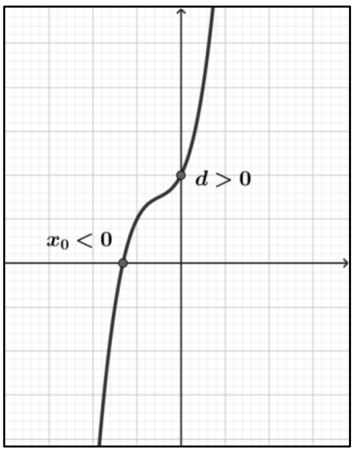

Figure 4

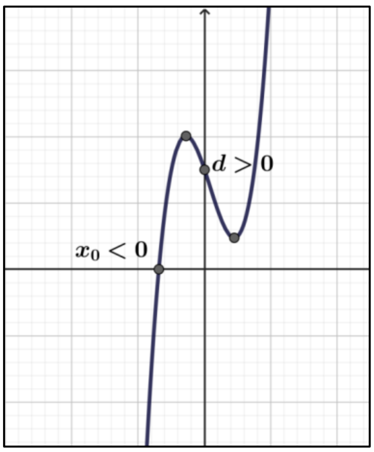

Figure 5

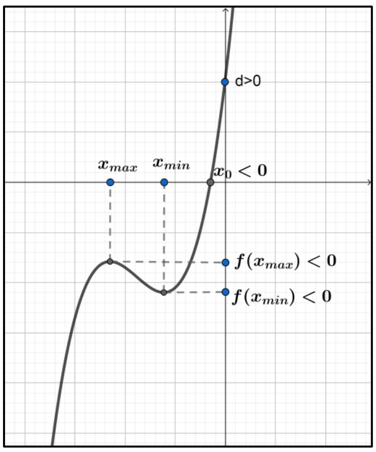

Figure 6

Theorem 2. For the cubic equation $ax^3 + bx^2 + cx + d = 0$ $(a > 0)$ to has the only negative real root, It is necessary and sufficient that $b^2 - 3ac \leq 0$, $d > 0$ or $b^2 - 3ac > 0$, $d > 0$ and the product of extrema values be positive.

Case 3. Consider the case where the cubic equation has 3 different real roots on the graph of the cubic function.

Figure 7 shows that the cubic function intersects the positive part of the *x*-*a*xis at 3 points. So, in this case, the cubic equation has 3 positive roots.

In Figure 8, the cubic function intersects the x- axis at 3 points, one of which intersects the positive part of the x-axis. So the cubic equation has 2 negative and 1 positive root.

From the graph depicted in Figures 7 and 8, it can be seen that the function has extrema, and from this it is clear that $\Delta = b^2 - 3ac > 0$. Since the extrema values are $f(x_{max}) > 0$, $f(x_{min}) < 0$, their product $f(x_{max}) \cdot f(x_{min}) < 0$ is negative.

The graph of the function crosses the negative part of the y-axis, i.e. $f(0) = a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d = d < 0$.

Summarizing the results, the following theorem is valid.

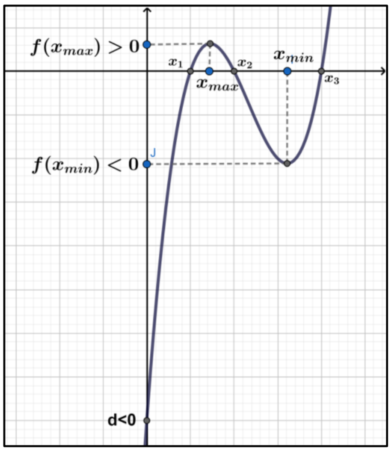

Figure 7

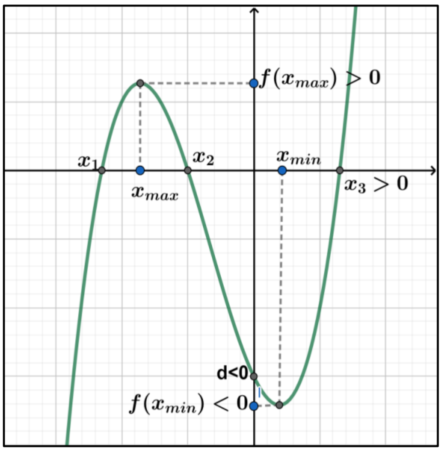

Figure 8

Theorem 3. In order for a cubic equation $ax^3 + bx^2 + cx + d = 0$ $(a > 0)$ to has three different real roots, and one or all three of them are positive so it is necessary and sufficient that $b^2 - 3ac > 0$, $d < 0$ and the product of extrema values are negative.

Case 4. The following theorem holds when a cubic equation has 3 negative real roots or 2 positive and 1 negative real roots.

Theorem 4. In order for a cubic equation $ax^3 + bx^2 + cx + d = 0$ $(a > 0)$ to has three different real roots, one or all three of which are negative, it is necessary and sufficient that$b^2 - 3ac > 0$, $d > 0$ and the product of extrema values are negative.

Case 5. Consider the case when the cubic equation has two equal roots on the graph of the cubic function.

In Figures 9-10, the extremum point of the cubic function and the zeros of the function are equal, and in this case, the cubic equation has same roots, i.e., $x_1 = x_2 = x_{max}$. Also, the extrema values $f(x_{max}) = 0$ and $f(x_{min}) < 0$, their product is $f(x_{max}) \cdot f(x_{min}) = 0$.

The graph of the function crosses the negative part of the y-axis, that is, $f(0) = a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d = d < 0$.

Based on the above conditions, the following theorem is valid.

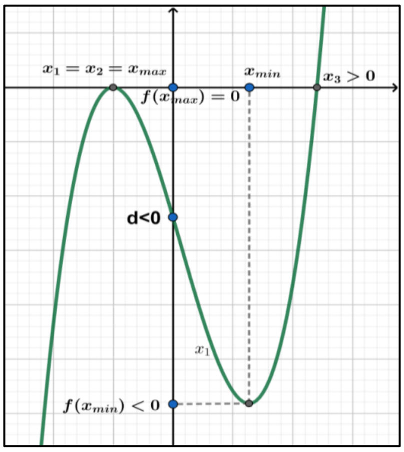

Figure 9

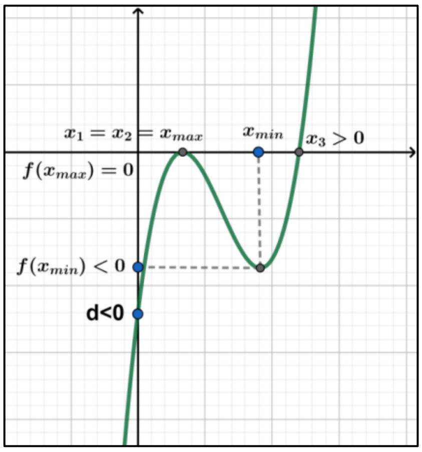

Figure 10

Theorem 5. For the cubic equation $ax^3 + bx^2 + cx + d = 0$ $(a > 0)$ to has one positive real root and two equal roots, it is necessary and sufficient that $b^2 - 3ac > 0$, $d < 0$ and the product of extrema values should be equal to 0.

Figures 11-12 show the graph of a cubic function when the cubic equation has one negative root and two equal roots.

The following theorem holds for a cubic equation to have two equal roots and one negative root.

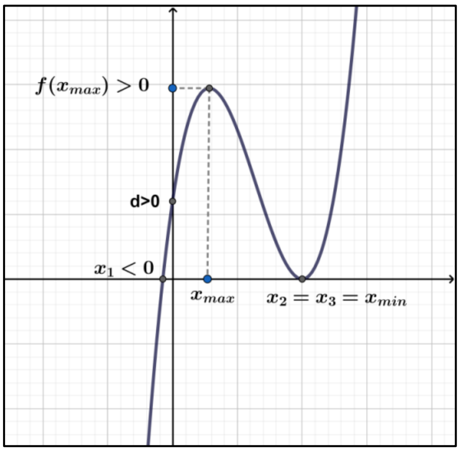

Figure 11

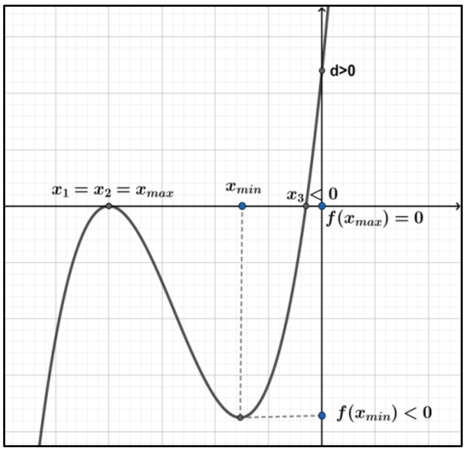

Figure 12

Theorem 6. In order for the cubic equation $ax^3 + bx^2 + cx + d = 0$ $(a > 0)$ to has one negative and two equal roots, it is necessary and sufficient $b^2 - 3ac > 0$, $d > 0$ and the product of extrema values are equal to 0.

In the theorems about the location of the roots of the cubic equation, the product of the extrema values of the function is important.

Function $f(x) = ax^3 + bx^2 + cx + d$ has extremum, when $b^2 - 3ac \geq 0$, i.e., when the discriminant of a square trinomial $3ax^2 + 2bx + c$ representing the derivative of this function is non-negative.

When $b^2 - 3ac < 0$, the quadratic equation has complex x_1 and x_2 roots. In this case, you can also look at the product $f(x_1) \cdot f(x_2)$.

When finding the product $f(x_1) \cdot f(x_2)$, it is not necessary to know the values of $f(x_1)$ and$f(x_2)$. The Vieta's formula can be used.

$x_1 x_2 = \frac{c}{3a}$; $x_1 + x_2 = \frac{-2b}{3a}$.

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = \frac{4b^2 - 6ac}{9a^2}.$$

$$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1 x_2 (x_1 + x_2) = \frac{-8b^3 + 18abc}{27a^3}.$$

Using the above formulas, we form the expression:

$$f(x_1) \cdot f(x_2) = (ax_1^3 + bx_1^2 + cx_1 + d)(ax_2^3 + bx_2^2 + cx_2 + d) =$$
$$= a^2(x_1x_2)^3 + b^2(x_1x_2)^2 + c^2(x_1x_2) + d^2 + ab(x_1x_2)^2(x_1 + x_2) +$$
$$+ac(x_1x_2)(x_1^2 + x_2^2) + ad(x_1^3 + x_2^3) + bc(x_1x_2)(x_1 + x_2) + bd(x_1^2 + x_2^2) +$$
$$+cd(x_1 + x_2) = \frac{-(b^2c^2 - 4ac^3 - 4b^3d - 27a^2d^2 + 18abcd)}{27a^2}.$$

The following expression is called the discriminant of a cubic equation:

$D = b^2c^2 - 4ac^3 - 4b^3d - 27a^2d^2 + 18abcd$

Thus, the discriminant of the cubic equation is completely determined by the product of the extreme values of the function corresponding to it.

CONCLUSIONS AND PERSPECTIVES FOR A FURTHER RESEARCH

Drawings and graphs do not prove the theorem, but they are important in making the theorem easy and understandable for the student, and in keeping it in his memory for a long time.

The use of drawings and graphs in solving mathematical problems allows to clearly understand the condition of the problem. At the same time, it allows to choose the correct method of solving the problem. In the condition of the problem, it is possible to clearly see the given situation in drawings and graphs, and thus it becomes easier to find a way to solve the problem to a certain extent. Because it is much easier to evaluate the visible situation than to draw a conclusion by reading only the text part without any visual aids.

In the article, theorems were developed and proved for the cubic equation to have a single real root, to have three different roots, and to have two equal roots. In this case, it was shown that the cubic equation has roots using the graph of the cubic function. Through this, it was shown that it is possible to develop visual thinking in students.

The above theorems can also be considered in the case $a < 0$. This can also be done using cubic function graphs.

We recommend studying the location of the roots of the cubic equation to students of upper grades of general education schools in maths club training.

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