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МЕТОДИКИ НАВЧАННЯ УЧНІВ  
МАТЕРІАЛУ ЗМІСТОВОЇ ЛІНІЇ  
«ФУНКЦІЇ ТА ЇХ ГРАФІКИ»

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<https://orcid.org/0000-0002-3131-1915>ON THE EXPEDIENCY AND DIRECTIONS  
OF IMPROVING THE METHODOLOGY  
OF TEACHING STUDENTS  
MATERIAL OF THE CONTENT LINE  
"FUNCTIONS AND THEIR GRAPHS"

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## АНОТАЦІЯ

**Постановка проблеми.** Вдосконалення методики навчання учнів матеріалу змістової лінії «Функції та їх графіки» наразі є актуальною проблемою, оскільки знання властивостей елементарних функцій сприяє формуванню в учнів адекватного світосприйняття на основі набутих загальних і предметних компетентностей. Функції мають численні практичні застосування та використовуються під час вивчення матеріалу шкільного курсу фізики, хімії, біології, інформатики тощо. Також матеріал змістової лінії «Функції та їх графіки» є досить важливим для успішного складання зовнішнього незалежного оцінювання якості знань з математики. Додатково потребу вдосконалення методики навчання учнів функціям підтверджують результати Міжнародного порівняльного дослідження PISA 2018, статистичні звіти Українського центру оцінювання якості освіти щодо тестування з математики та опитування вчителів математики.

**Матеріали та методи.** Для досягнення мети ми використовуємо теоретичний метод аналізу методичної літератури з досліджуваного питання. Ми також використовуємо деякі емпіричні методи: власне опитування за допомогою Google forms, спостереження за навчальним процесом у загальноосвітніх школах, аналіз навчальних досягнень учнів. У цій статті ми також використовуємо комплекс методів наукового пізнання: порівняльний аналіз для з'ясування різних поглядів на проблему; систематизація та узагальнення з метою зробити висновки та сформулювати рекомендації щодо напрямків вдосконалення методики навчання учнів матеріалу змістової лінії «Функції та їх графіки»; узагальнення педагогічного досвіду та спостережень авторів.

**Результати.** Сформульовано гіпотезу щодо доцільності вдосконалення методики навчання учнів загальноосвітніх шкіл матеріалу змістової лінії «Функції та їх графіки». З метою підтвердження цієї гіпотези та окреслення напрямків такої вдосконалення з використанням сучасних інформаційно-комунікаційних технологій проведено опитування вчителів. Опитування проводилося анонімно і онлайн з використанням Google forms. У опитуванні взяли участь близько 100 вчителів, які мають різний педагогічний стаж та кваліфікацію, але більшість із них працюють у школі більше 15 років. Опитування показало, що більшість респондентів часто або постійно використовують у навчальному процесі сучасні технології, а також переконані в необхідності вдосконалення методики навчання учнів матеріалу змістової лінії «Функції та їх графіки». Одним із напрямків такого вдосконалення вони вбачають використання сучасних технологій на уроках математики, а також розробку відповідних дидактичних матеріалів і методичних рекомендацій щодо їх використання.

## ABSTRACT

**Formulation of the problem.** Improving the methodology of teaching students the material of the content line "Functions and their graphs" is currently a relevant problem, because knowledge of the properties of elementary functions contributes to the formation of students' adequate worldview based on acquired general and subject competencies. The functions have a lot of practical applications and are used during studying the material of the school course of physics, chemistry, biology, computer science, etc. Also, the material of the content line "Functions and their graphs" is very important for the successful compilation of external independent assessment of the quality of knowledge in mathematics. In addition, the need to improve the methodology of teaching students functions is confirmed by the results of the International Comparative Survey PISA 2018, statistical reports of the Ukrainian Center for Educational Quality Assessment on mathematics testing, and surveys of mathematics teachers.

**Materials and methods.** To achieve our goal, we use the theoretical method of analysis of the methodological literature on the subject. We also use some empirical methods: our own survey using Google forms, monitoring the learning process in secondary schools, and analysis of student achievement. In this article, we also use a set of methods of scientific knowledge: comparative analysis to clarify different views on the problem; systematization, and generalization in order to draw conclusions and formulate recommendations for improving the methodology of teaching students the material of the content line "Functions and their graphs"; generalization of pedagogical experience and observations of the authors.

**Results.** The hypothesis concerning the expediency of improvement of a technique of training of students of secondary schools of a material of the content line "Functions and their graphs" is formulated. In order to confirm this hypothesis and outline the directions of such improvement with using of modern information and communication technologies, a survey of teachers has been conducted. The poll has been conducted anonymously and online using Google forms. About 100 teachers with different teaching experience and qualifications took part in the survey, but most of them have been working at the school for more than 15 years. The survey showed that most respondents often or constantly use modern computer technologies in the educational process, and are convinced of the need to improve the methodology of teaching students the material of the content line "Functions and their graphs". One of the directions of such improvement, they believe the use of modern computer technologies in mathematics lessons, as well as the development of appropriate didactic materials and guidelines for their application.

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**Висновки.** Для вдосконалення методики навчання учнів матеріалу змістової лінії «Функції та їх графіки» природним видається використання сучасних інформаційних технологій. Учителі наразі активно застосовують сучасні електронні засоби для навчання математики, але потребують методичної підтримки щодо їх використання в навчальному процесі. Зокрема, це стосується і вивчення матеріалу змістової лінії «Функції та їх графіки»: є потреба в розробці готових дидактичних матеріалів та у створенні методичних рекомендацій щодо використання інформаційних технологій для вивчення матеріалу цієї змістової лінії, які дозволитимуть застосовувати сучасні електронні засоби в навчальному процесі більш ефективно.

**КЛЮЧОВІ СЛОВА:** методика навчання математики; компетентнісний підхід; функції та їх графіки; інформаційно-комунікаційні технології; комп'ютерні технології; електронні засоби навчання.

**Conclusions.** To improve the methodology of teaching students the material of the content line "Functions and their graphs" it seems natural to use modern computer technologies. Teachers are currently actively using modern electronic tools for teaching mathematics, but need methodological support for their application in the educational process. In particular, this applies to studying the content line "Functions and their graphs": there is a need to develop ready-made didactic materials and to create guidelines for using information technology to study the material of this content line, which will use modern computer tools in the educational process more effectively.

**KEYWORDS:** methodology of teaching mathematics; competency approach; functions and their graphs; information and communication technologies; computer technologies; electronic learning tools.

## INTRODUCTION

**Formulation of the problem. Analysis of current research.** Educational activities, both in general and in the field of mastering mathematics, is a difficult but interesting job. To achieve results during this work, students need time, a qualified teacher, as well as significant own effort. In the process of learning, students have to acquire the necessary competencies for life, i.e. learn to think, understand the essence of things, comprehend ideas and on this basis to seek the necessary information, interpret them, forming information about objects and processes of the real world and the world of abstract ideas. Also students have to acquire knowledge, skills and abilities in specific conditions, to form and defend one's own opinion.

The role of the teacher in the learning process is to constantly monitor the activities of students in order to provide timely assistance in overcoming difficulties in the process of acquiring and consolidating appropriate competencies. Modern reality puts before teachers the task of updating and replenishing their knowledge on a wide range of issues, which together should become the content of educational and methodological work and help students cope with problems in learning. At the same time, it is often quite difficult for a teacher to evaluate, form criteria and indicators of personal growth of students. Therefore, during preparing for lessons, it is important to take into account possible students' mistakes and correct them in time.

Modern Ukrainian education is aimed at personality-oriented learning (Hryniova, 2015). Therefore, the teacher has a task to reveal, preserve and develop the individuality of each student, to identify and take into account his potential, needs, desires and, accordingly, to provide a differentiated approach to learning. This approach requires the creation in the learning process of a modern educational environment that will promote the development of students' necessary skills and competencies (learning ability, creativity, critical thinking, communication, teamwork, etc.), the formation of his personality. To do this, the teacher can use not only traditional teaching aids, but also modern educational trends, in particular, STEM education, partnership pedagogy, personalization of learning, inclusive learning, practical training, research training, gamification, online courses, etc. (Vasylyeva, 2020; Lantarón et al., 2021).

Traditionally, during studying mathematics at school, all the material is divided into so-called content lines, the development and deepening of which accomplish throughout the learning process by considering the relevant topics in each class (MES of Ukraine, 2017). In Ukrainian methodology tradition the following content lines are considered: "Numbers and expressions", "Functions and their graphs", "Equations and inequalities", "Geometry on a plane", "Geometry in the space", "Stochastics" (UCEQA, 2016). Each of these content lines is important and, perhaps, it is impossible to single out any of the most important of them. However, the study of some of them begins in the 1st grade and ends in the final grade, and for others the study begins a bit later, but continues until the end of training. For example, the study of the content line "Numbers and expressions" begins with actions on natural numbers in primary school, and ends with the study of irrational and transcendental numbers in senior school. At the same time, the semantic line "Functions and their graphs", if we do not take into account the propaedeutic material, begins to be studied from 7th grade on the topic "Linear function" and ends in the final grade by studying the topic "Exponential and logarithmic functions". In this article, we will take a closer look at the methodology of studying the material of the content line "Functions and their graphs" in the school course of mathematics.

Note that improving the methodology of teaching students the material of this content line is currently an urgent problem, as evidenced by the publications of foreign and domestic scholars (Bakker et al., 2021; Vasylyeva, 2020; Bakhruhshyn, 2019). We also emphasize that knowledge of the properties of elementary functions contributes to the formation of students' adequate worldview on the basis of acquired general and subject competencies. Without appropriate knowledge, skills and abilities, it is difficult to imagine the study of many topics in physics, chemistry, biology, computer science, etc. (Andrews, 1998; Goudriaan & Monteith, 1990; Martínez-Legaz et al., 2005; Boucekkine & Ruiz-Tamarit, 2008). Also, the material of the content line "Functions and their graphs" is very important for the successful cope of external independent assessment (EIA) of the quality of educational achievements in mathematics, which is conducted by the Ukrainian Center of Educational Quality Assessment (UCEQA). In particular, about 20% of the total number of EIA test tasks in mathematics relate to functions and their graphs.

Based on our own pedagogical experience and the results of the state final attestations (SFA) in mathematics, we found problems in the relevant understanding of students of grades 10-11 material of the content line "Functions and their graphics". This is also stated in the final reports of the UCEQA during the analysis of test results in mathematics. For example, it is noted that: "A significant part of the participants of the external independent assessment do not have developed basic skills on the topic "Functions. Properties of functions". Test participants often could not properly analyze and understand the information presented in graphical form (on the figure), compare it with the textual part of the condition of the task, to build an appropriate mathematical model. Mostly, this is applied to problems of practical content in both algebra and geometry" (UCEQA, 2017).

An analysis of the official reports of the UCEQA for 2016-2021 on the presence of student problems with the study of functions (UCEQA, 2016; UCEQA, 2017; UCEQA, 2018; UCEQA, 2019; UCEQA, 2020; UCEQA, 2021) showed the following:

- students do not have the skills to analyze and understand the data presented using a graph or figure during comparing them with the condition of the task (mostly this applies to tasks for recognizing graphs of functions);
- half of the test participants cannot determine the point of intersection of the graph of the function with the abscissa axis in the MCQ type of tasks;
- students had difficulties in solving the problem of establishing correspondence between the graphs of functions shown in the figures and the properties of these functions;
- students cannot correctly find the domain of the function and calculate the value of the function at given points in the domain;
- students cannot correctly draw the function graph in the open-ended tasks (tasks with a full explanation);
- students cannot correctly determine the derivative of the function and from these data to find the intervals of monotonicity and extremum points of the function;
- students do not understand the geometric sense of a definite integral and make mistakes during determining the antiderivative function and during calculating the area of a geometric figure using the Newton-Leibniz formula;
- only from 1.7% to 9.4% of test participants were able to fully cope with the open-ended tasks, and from 40.3% to two thirds of participants did not complete these tasks at all.

Analysis of statistical characteristics of test tasks related to the material of the content line "Functions and their graphs" shows that test participants in mathematics on EIA insufficiently and fragmentarily substantiate the conclusions, often cannot coherently, logically and consistently describe in written form the way of their thoughts, cannot use acquired knowledge and skills for analysis and synthesis of the obtained results (UCEQA, 2016).

In 2018, Ukraine participated in the International Comparative Study PISA, during which it was found that the average level of mathematical literacy of Ukrainian students (453 points) corresponds to the second level out of six possible (Bakhrushyn, 2019). This level is basic, reflecting the ability of students to participate effectively and productively in society. PISA scores are determined on scales specifically designed for each field of study (for reading, mathematics and natural sciences). Scales are a numerical indicator of the levels of literacy of students in the appropriate fields. For each level there is a certain number of test tasks, where the complexity of tasks at each level gradually increases. The amount of points received by the student allows to determine the level of his competence in each of the disciplines. The results of PISA participants in each field are presented on a scale, which identifies six levels of literacy in each of the subject areas of assessment (Mazorchuk et al., 2019; Vakulenko et al., 2021).

Comparing Ukraine and other countries participating in the study, it was found that in Ukraine the most problematic among the three branches of PISA is mathematics, noticeable differences are in different subject areas, especially noticeable relatively low are the students results in mathematics. 36% of Ukrainian students failed to demonstrate such a level, of which 15.6% did not even reach the first level of mathematical literacy. At the same time, about 38% of Ukrainian test participants showed the results of 3-6 levels, but only a few of them reached the sixth level (Bakhrushyn, 2019).

The PISA study on content line "Functions and their graphs" focuses on linear functions, their properties, various forms of description and tasks. The next PISA study, scheduled for 2022, will look more closely at the relationship between quantities, which can be expressed through equations, graphs, tables, or verbal descriptions. The concept of a function as such, i.e. as such an abstract object, the representations of which are various equations, graphs, tables or descriptions, has been studied. Two views of a function — a naive view of the process and a more abstract view of the object — can be reconciled in the graph of the function. However, the analysis of the graph of the function, determining the coordinates of the values on the axes are also dynamic or process nature. The graph of a function is an important tool for studying the concept of rate of change. The graph appears as a kind of visual means for understanding the function as a relationship between the related quantities (Vakulenko et al., 2021). It is this study that indicates the existence of the problem of studying the content line of "Functions and their graphs", shows the subject connections of this line and the relevance of its study.

Summing up, we see that students of Ukrainian schools have significant problems with mastering the material of the content line "Functions and their graphs". Only the fact that according to the statistical reports of the UCEQA in 2016-2021, the task of establishing a correspondence between the function and the property of the function is performed correctly by only 16.2% to 28.8% of participants in the external examination in mathematics, indicates an urgent need to improve methodology of teaching students the material of this content line.

*The purpose of the article* is to find out the expediency and possibility of improving the methodology of teaching students the material of the content line "Functions and their graphs" and to outline the directions of such improvement, based on using of modern computer teaching aids in mathematics.

## RESEARCH METHODS

To achieve our goal, we use the theoretical method of analysis of the methodological literature on the subject. We also use some empirical methods: our own survey using Google forms, monitoring the learning process in secondary schools, analysis of students' achievement. In this article we also use a set of methods of scientific knowledge: comparative analysis to clarify different views on the problem; systematization and generalization in order to draw conclusions and formulate recommendations on how to improve the methodology of teaching students the material of the content line "Functions and their graphs"; generalization of pedagogical experience and observations of the authors.

## RESULTS OF RESEARCH

The above theoretical analysis provides sufficient grounds for formulating a hypothesis about the feasibility of improving the teaching methodology of the content line "Functions and their graphs" for secondary school students. In order to confirm (or refuse) this hypothesis and outline the directions of such improvement with using of modern information and

communication technologies (ICT), we conducted a survey of teachers. The poll has been conducted anonymously and online using Google forms. It involved about 100 teachers working with grades 5-11 of secondary schools in Ukraine, with different teaching experience and qualifications. However, as it turned out, most of them are experienced professionals - 51% of respondents have been working as mathematics teachers more than 15 years.

Below we put the list of questions of survey and the results of this poll. If the question contained answer options, then after each of these options in parentheses is the percentage of the total number of respondents who chose this option.

*Question 1.* Do you use modern ICT in your professional activity?

- Yes, all the time (43%).
- Yes, often (46%).
- Yes, sometimes (10%).
- Yes, but very rarely (1%).
- No, I don't use it at all (0%).

As we can see, all surveyed teachers use ICT in different ways in their work, and the vast majority of them (91%) use them constantly and regularly. This situation is natural, especially in the context of a significant spread of distance and blended learning due to the onset of the pandemic situation and martial law in Ukraine.

*Question 2.* What software (electronic) tools do you use to teach students (you could choose several answers)?

- Electronic control systems of knowledge, skills and abilities (test aggregators) (86%).
- Electronic textbooks of various types (from pdf versions to interactive) (77%).
- Demonstration tools (videos, text presentations, etc.) (81%).
- Training organization systems (training platforms) (64%).
- Data search systems (electronic libraries, Internet search engines, etc.) (58%).
- Software for learning in the form of games (quizzes, competitions, puzzles, etc.) (53%).
- Programs that allow modeling experiments, imaginary or real life situations, etc. (23%).
- Programs that provide the opportunity to create new electronic resources (17%).

The results of the answers to this question show that the most popular electronic tools for teachers are programs that allow to assess students' academic achievements, as well as electronic learning tools, such as textbooks, instructional videos and presentations. Less popular, but quite common are systems for organizing training and data retrieval, as well as programs for learning in the form of games. In our opinion, this is due to the fact that test aggregators and e-textbooks can be used both during classroom learning and during distance and blended learning, and the rest of the mentioned e-learning tools are more typical for online lessons.

*Question 3.* Sort by importance (1 is the most, on your opinion, important, 6 is the least important) the following content lines of the school course of mathematics: "Numbers and expressions", "Functions and their graphs", "Equations and inequalities", "Geometry on a plane", "Geometry in the space", "Stochastics".

The table below shows the percentage from the total number of teachers surveyed who ranked each of these content lines from 1 to 6 in descending order of importance. The far right column contains the weighted average place for each content line.

Content line / place	1	2	3	4	5	6	Weighted average place
<i>Numbers and expressions</i>	72%	6%	9%	4%	4%	5%	1,8
<i>Function and their graphs</i>	9%	14%	40%	17%	10%	10%	3,3
<i>Equations and inequalities</i>	9%	60%	13%	5%	9%	4%	2,6
<i>Geometry on a plane</i>	3%	17%	25%	44%	8%	3%	3,5
<i>Geometry in the space</i>	4%	4%	4%	16%	49%	23%	4,7
<i>Stochastics</i>	5%	1%	9%	13%	17%	55%	5,0

The weighted average place shows that the content line "Functions and their graphs", according to teachers, is the third most important after the content lines "Numbers and expressions" and "Equations and inequalities". 40% of respondents put it in this place. This position is rather natural, because without proper knowledge of numbers and expressions, the study of mathematics is fundamentally impossible, and equations and inequalities are the most popular models of real phenomena and processes. However, this does not mean that teachers underestimate the importance of functions, because much more teachers put it on the 1-3 places than in 4-6 places (63% vs 37%). Thus, the study of functions and their graphs respondents consider, though not the most important, but one of the priorities in the study of school mathematics course.

*Question 4.* Do you suppose it appropriate to improve the methodology of teaching students the content line "Functions and their graphs"?

- Yes, definitely (22%).
- More likely, yes (55%).
- It is difficult to answer (10%).
- More likely, no (12%).
- No, definitely (1%).

As we predicted by our theoretical research, a total of 77% of respondents are convinced of the need to improve the methodology of teaching students the content line "Functions and their graphs". At the same time, it is important that 82% of

experienced teachers with 15 years of experience or more suppose the same. This is further evidence that with experience of teaching, awareness of the need to improve the methodology of learning functions is only growing.

*Question 5.* Are you currently provided with appropriate methodological materials for teaching students the content line "Functions and their graphs"?

- Yes, definitely (5%).
- More likely, yes (48%).
- It is difficult to answer (21%).
- More likely, no (25%).
- No, definitely (1%).

The number of teachers who believe that they are provided with learning materials on the study of functions is about half of the total number of respondents. The rest of the teachers feel the need for such materials, although none of them stated that there were no such materials at all. This suggests that the situation with the methodological support of teachers to study the content line "Functions and their graphs" is not catastrophic and needs only minor improvements.

*Question 6.* If you do not have enough methodological materials to teach students the content line "Functions and their graph", then which ones (you could choose several answers)?

- Educational demonstration materials (videos, text presentations) (46%).
- User instructions on software capabilities (46%).
- Methodical advice on using of software to teach students the material of the content line (44%).
- Ready-made developments for monitoring of students' achievement (tests, control works, etc.) (38%).
- Electronic manuals (textbooks, collections of tasks, etc.) (31%).
- Materials for in-depth study (1%).
- Fully provided with methodical materials (5%).

As we can see, during studying the content line "Functions and their graphs" teachers are most lacking in educational demonstration materials, there are some problems with user instructions on the possibilities of teaching software, as well as ready-made tests current and final control works. Note that only 5% of respondents in response to question 5 said that they are fully provided with methodological materials related to this content line. This confirms our confidence in the need for further methodological developments to study the functions, and is fully coincident with the answers to question 4.

*Question 7.* If you think that you need to improve the method of teaching students the content line "Functions and their graphs", then what can be done for it (you could choose several answers)?

- Develop guidelines for studying the material of this content line using ICT (64%).
- Increase the number of hours to study the material of this content line (55%).
- Increase the number of ready-made educational materials (electronic textbooks, videos, etc.) (36%).
- Develop methodological recommendations for improving the methodology of traditional teaching of the material of this content line (26%).
- Create a learning environment with an existing systematized database of materials and opportunities for control and self-control (1%).

The answers to this question show that half of the teachers surveyed are in favor of an extensive way to ensure the assimilation of the necessary material and want to increase the number of hours to study the content line. However, this path is obviously a dead end, as increasing study time does not guarantee a positive result. In addition, the content line "Functions and their graphs" cannot be called the most important among all others, and therefore, the increase in the amount of study time on it is questionable for this reason.

However, it is gratifying that many more teachers understand the futility of the extensive path and choose the intensive one - improving guidelines for studying functions (both traditional methods and using ICT) (see more also in Birgin & Uzun Yazıcı, 2021 and Öçal, 2017). In this intensive way of ensuring the quality of education, the need to increase the number of ready-made teaching materials with using of modern technologies, which is emphasized by the interviewed teachers, seems natural.

Summarizing all the survey data, we can say that:

- most modern mathematics teachers, and especially teachers with much experience in school, constantly use ICT in their educational activities;
- the range of use of ICT in teaching mathematics is quite wide - from electronic teaching aids (textbooks, educational videos, etc.) to means of monitoring the results of educational activities;
- the content line "Functions and their graphs" is not, on the opinion of teachers, the most important content line of the school course of mathematics, but its importance is undeniable;
- teachers have no doubts about the need to improve the methodology of teaching students the material of the content line "Functions and their graphs", as well as the need to develop appropriate methodological materials, in particular, using modern computer technologies.

## CONCLUSIONS AND PROSPECTS OF FURTHER RESEARCH

According to the analysis of scientific and methodological literature, in particular, the results of EIA and SFA in recent years, as well as the international comparative study PISA 2018, in Ukraine there is a problem of ensuring the proper quality of student knowledge, skills and competencies for the material of the content line "Functions and their graphs". The existence of such a problem is confirmed by our survey of mathematics teachers, the results of which are given above.

To improve the methodology of teaching students the material of the content line "Functions and their graphics" it seems natural to use computer technologies. According to the above-mentioned survey, teachers are currently actively using modern electronic tools for teaching mathematics, but need methodological support for their use in the educational process. In

particular, this applies to the study of functions and their graphs: there is a need to develop ready-made didactic materials and to create guidelines for using ICT to study the material of this content line, which will allow using of modern electronic tools in the learning process more effectively.

On our opinion, this direction of research (using of modern electronic aids in the study of the content line "Functions and their graphs") is promising and necessary for teachers. In the future, we plan a series of publications that will address detailed guidelines for the use of ICT to study certain types of functions (linear, quadratic, power, exponential, logarithmic, trigonometric) in the school course of mathematics.

We also consider the development of relevant online courses, trainings and webinars for mathematics teachers, as well as their placement on educational platforms such as Prometheus, Osvitoria, Vseosvita, GIOS, etc. to be promising. We are convinced that the availability of such educational resources, as well as a large number of e-learning tools will help overcome the problems covered in the study of the content line "Functions and their graphs", make the learning process more effective for both teachers and students.

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